5/25/17 "Live as if you are going to die tomorrow, learn as if you are going to live forever." -Ghandi

HW: Final Review #6-10
Final on Wednesday 6/7

1) $\text{Length} = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

$f(x) = x^2$

$f'(x) = 2x$

$\text{Length} = \int_0^2 \sqrt{1 + (2x)^2}\, dx = 4.647$

$g(x) = x^3$

$g'(x) = 3x^2$

$\text{Length} = \int_0^2 \sqrt{1 + (3x^2)^2}\, dx = 8.630$

$y = x^3$ is longer over $[0, 2]$

2

2) Antiderivative:

a) $\int 3x^4 + 2 \, dx$

$$\boxed{f(x) = \frac{3x^5}{5} + 2x + c}$$

b) $\int (5\sqrt{x} - 3x) \, dx$

$$\int \left(5x^{\frac{1}{2}} - 3x\right) dx$$

$$f(x) = \frac{5x^{3/2}}{3/2} - \frac{3x^2}{2} + C$$

$$= \frac{10}{3}x^{3/2} - \frac{3}{2}x^2 + c$$

$$= \boxed{\frac{10}{3}\sqrt{x^3} - \frac{3}{2}x^2 + c}$$

3

3) a) $\int_0^{\frac{\pi}{2}} \cos(x)\,dx$

$= \sin x \Big]_0^{\frac{\pi}{2}}$

$= \sin\left(\frac{\pi}{2}\right) - \sin(0)$

$1 - 0$

$= \boxed{1}$

b) $\int_1^2 (3x + x^3)\,dx$

$= \frac{3x^2}{2} + \frac{x^4}{4} \Big]_1^2$

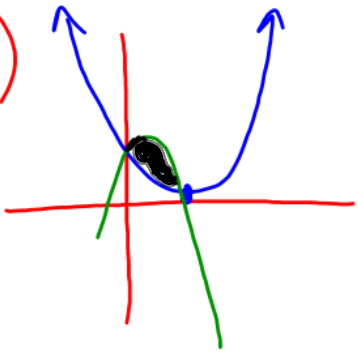
$\left(\frac{3(2)^2}{2} + \frac{2^4}{4}\right) - \left(\frac{3(1)^2}{2} + \frac{1^4}{4}\right)$

$(6 + 4) - \left(\frac{3}{2} + \frac{1}{4}\right)$

$10 - \frac{7}{4}$

$\boxed{\frac{33}{4}}$

4)



NORMAL FLOAT AUTO REAL DEGREE MP

$\int_0^2 (-X^2+4-((X-2)^2))dX$

2.666666667

Ans▸Frac

$\frac{8}{3}$

5) $y = x^2 - 4$ $\quad y = 0$ (x-axis)

$V = \pi \int_a^b f(x)^2 \, dx$

NORMAL FLOAT AUTO REAL DEGREE MP

$\int_{-2}^2 ((X^2-4)^2)dX$

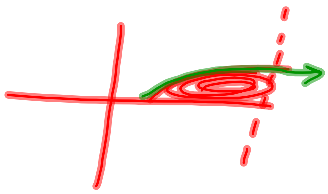
34.13333333

Ans▸Frac

$\frac{512}{15}$

$\boxed{V = \frac{512}{15}\pi \text{ units}^3}$

6) $y = \sqrt{x-2}$ $\quad y = 0$ $\quad x = 5$



$\text{Volume} = \pi \int_a^b f(x)^2 \, dx =$

NORMAL FLOAT AUTO REAL DEGREE MP

$\int_2^5 ((\sqrt{X-2})^2)dX$

4.5

$4.5\pi \text{ units}^3$

7) Slope $= 6x^2-2x+3$

Point $(1,3)$ x y

$$\int 6x^2-2x+3 = \frac{6x^3}{3}-\frac{2x^2}{2}+3x+c$$

$2x^3-x^2+3x+c$

$3=2(1)^3-1^2+3(1)+c$

$3=2-1+3+c$

$$\boxed{f(x)=2x^3-x^2+3x-1}$$

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$-1=c$

8) $y^2-2x=3$ what is $\frac{dy}{dx}$ @ $(3,3)$

$\frac{d}{dx}\left(y^2-2x=3\right)$ Implicit

derivative

evaluate derivative

$2y\frac{dy}{dx}-2=0$

$$\frac{2y\frac{dy}{dx}}{2y}=\frac{2}{2y} \longrightarrow \frac{dy}{dx}=\frac{2}{2y}=\frac{1}{y}$$

Evaluate @ $(3,3)$

$$\left.\frac{dy}{dx}\right|_{(3,3)}=\frac{1}{3}$$

9) a) $\lim_{x \to 0} \frac{24x^3 - 3x}{12x^3 + x} = \frac{24(0)^3 - 3(0)}{12(0)^3 + (0)} = \frac{0}{0}$ Ind.

Try to reduce

Try to plug in again

$$= \frac{3\cancel{x}(8x^2 - 1)}{\cancel{x}(12x^2 + 1)} = \frac{24x^2 - 3}{12x^2 + 1} = \frac{24(0)^2 - 3}{12(0)^2 + 1} = \frac{-3}{1}$$

$\lim_{x \to 0}$ is -3

b) $\lim_{x \to \infty} \frac{24x^{3} - 3x}{12x^{3} + x}$

degrees	limit
N > D	∞ or $-\infty$
D > N	0
N = D	coefficients

Look at degrees (highest exponents)

$\frac{24}{12} = \boxed{2}$

10) $y = \dfrac{x+2}{2x-1}$ tangent when $x = 3$

need Slope

Quotient Rule

$$y' = \frac{(2x-1)(1)-(x+2)(2)}{(2x-1)^2}$$

$$y'_{@x=3} = \frac{(2(3)-1)(1)-(3+2)(2)}{(2(3)-1)^2} = \frac{-5}{25}$$

Slope of tangent $= -\frac{1}{5}$

Point:

$x=3$ $\quad y = \dfrac{3+2}{2(3)-1} = \dfrac{5}{5} = 1$

$(3,1)$

Equation of tangent line @ $x=3$

$$\boxed{y-1 = -\frac{1}{5}(x-3)}$$

(*) Normal line is exactly the same with negative reciprocal slope.

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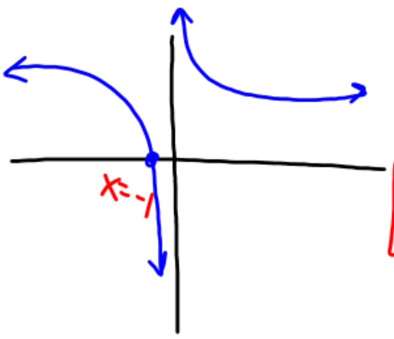
11) Concave down, the 2nd derivative is negative.

$$y = 6x^2 + \frac{x}{2} + 3 + \frac{6}{x}$$



Concave down between -1 and 0

$$\boxed{-1 < x < 0}$$

12) $f(x) = \frac{x^2-1}{2x}$ $\begin{matrix} f \\ g \end{matrix}$

Product Rule:

$f'g + fg'$

$$f'(x) = \frac{2x(2x) - (x^2-1)(2)}{(2x)^2}$$

Quotient Rule:

$$\frac{gf' - fg'}{(g)^2}$$

$$= \frac{4x^2 - 2x^2 + 2}{4x^2} = \frac{2x^2+2}{4x^2} = \boxed{\frac{x^2+1}{2x^2}}$$

13) Position $x(t) = \frac{1}{2}\sin(t) + \cos(2t)$

Velocity $x'(t) = \frac{1}{2}\cos(t) - 2\sin(2t)$

Acceleration $x''(t) = -\frac{1}{2}\sin(t) - 4\cos(2t)$

$$x''\left(\frac{\pi}{2}\right) = -\frac{1}{2}\sin\left(\frac{\pi}{2}\right) - 4\cos\left(\cancel{2}\frac{\pi}{\cancel{2}}\right)$$

$$= -\frac{1}{2}(1) - 4(-1)$$

$$= \boxed{3.5}$$

14) $f(x) = e^x \cdot \ln x$

Recall

$f(x) = e^x \quad f'(x) = e^x$

$f(x) = \ln x \quad f'(x) = \frac{1}{x}$

$f'(x) = e^x \cdot \frac{1}{x} + e^x \ln x$

$f'(e) = e^e \cdot \frac{1}{e} + e^e \underbrace{\ln e}$

$\ln_e e = 1$

$= \frac{e^e}{e} + e^e(1) \approx \boxed{20.729}$

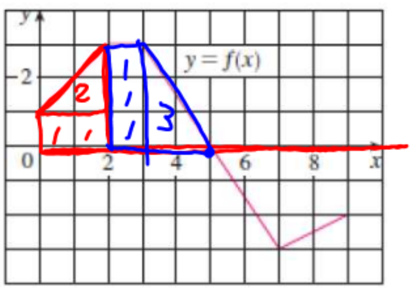
$$\text{Area} = \int_a^b \text{Top} - \text{bottom} \; dx$$

15. The graph of f is shown. Evaluate each integral by interpreting in terms of areas.

a. $\int_0^2 f(x)dx$ Area between f(x) and x-axis between 0 and 2

b. $\int_2^5 f(x)dx$

c. $\int_0^5 f(x)dx$



a) = 4

b) = 6

c) 4+6 = 10

7) Derivative $= 6x^2 - 2x + 3$
(slope)

Looking for Antiderivative

$$y' = 6x^2 - 2x + 3$$

$$y = \int 6x^2 - 2x + 3 \, dx$$

$$y = \frac{6x^3}{3} - \frac{2x^2}{2} + 3x + c$$

$$y = 2x^3 - x^2 + 3x + c$$

Passes through $(1,3)$

$$3 = 2(1)^3 - 1^2 + 3(1) + c$$

$$3 = 2 - 1 + 3 + c$$

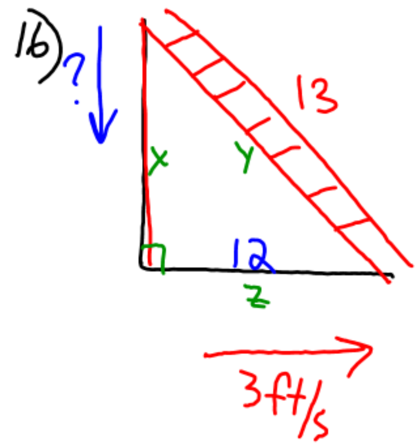
$$3 = 4 + c$$

$$-4 \quad -4$$

$$-1 = c$$

$$\boxed{y = 2x^3 - x^2 + 3x - 1}$$

16)



Know

$y = 13 \quad \frac{dy}{dt} = 0$

$z = 12 \quad \frac{dz}{dt} = 3$

$x = 5$

Want:

$\frac{dx}{dt}$

Find x:

$x^2 + 12^2 = 13^2$

$x = 5$

$$x^2 + z^2 = y^2$$

$$2x\frac{dx}{dt} + 2z\frac{dz}{dt} = 2y\frac{dy}{dt}$$

$$2(5)\frac{dx}{dt} + 2(12)(3) = 2(13)(0)$$

$$10\frac{dx}{dt} + 72 = 0$$

$$10\frac{dx}{dt} = -72$$

$$\frac{dx}{dt} = \frac{-72}{10} = -7.2 \text{ ft/sec}$$

Top of ladder is sliding down the wall @ 7.2 ft/sec

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18) $\dfrac{2(x+h)^2-8(x+h)+5-(2x^2-8x-5)}{h}$

$$\frac{\cancel{2x^2}+4xh+2h^2\cancel{-8x}-8h\cancel{+5}\cancel{-2x^2}\cancel{+8x}\cancel{+5}}{h}$$

$$\frac{4xh+2h^2-8h}{h}=\frac{\cancel{h}(4x+2h-8)}{\cancel{h}}=4x+2h-8$$

$h\to 0$

$\boxed{4x-8}$

19) $= \lim_{h \to 0} \dfrac{\overset{f(x+h)}{\sqrt{3(x+h)+1}} \overset{-f(x)}{- \sqrt{3x+1}}}{h} \cdot \dfrac{\sqrt{3(x+h)+1} + \sqrt{3x+1}}{\sqrt{3(x+h)+1} + \sqrt{3x+1}}$

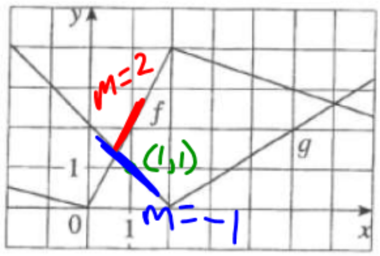
$\dfrac{3(x+h)+1-(3x+1)}{h\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)} = \dfrac{\cancel{3x}+3\cancel{h}\cancel{+1}-\cancel{3x}\cancel{-1}}{\cancel{h}\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)}$

$\lim_{h \to 0} = \dfrac{3}{\sqrt{3(x+h)+1}+\sqrt{3x+1}} = \dfrac{3}{\sqrt{3x+1}+\sqrt{3x+1}} = \dfrac{3}{2\sqrt{3x+1}}$

20. If f and g are the functions shown below. Let $h(x) = f(g(x))$ and $s(x) = f(x)g(x)$.

Chain Product

Find: $h'(1)$ and $s'(1)$



$g(1) = 1$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(1) \cdot g'(1)$

$h'(1) = 2 \cdot (-1) = -2$

$s'(x) = f(x) \cdot g'(x) + f'(x) g(x)$

$s'(1) = f(1) \cdot g'(1) + f'(1) \cdot g(1)$

$= (2) \cdot (-1) + (2)(1)$

$= -2 + 2$

$= \boxed{0}$

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2	5	4	3	4
3	0	6	-1	-2

quotient. chain

If $n(x)=\frac{f(x)}{g(x)}$, $h(x)=f(g(x))$, find the value of each of the following: a) $n'(2)$ b) $h'(1)$

a) $n'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$

$n'(2)=\frac{g(2)\cdot f'(2)-f(2)g'(2)}{(g(2))^2}=\frac{(3)(4)-(5)(4)}{3^2}$

$=-\frac{8}{9}$

b) $h'(x)=f'(g(x))\cdot g'(x)$

$h'(1)=f'(g(1))\cdot g'(1)$

$=f'(2)\cdot(3)$

$=(4)(3)$

$\boxed{h'(1)=12}$

18

approaching 1 from left

22. $\lim_{x\to 1^-} f(x) = -\infty$

23. $\lim_{x\to 1^+} f(x) = \infty$

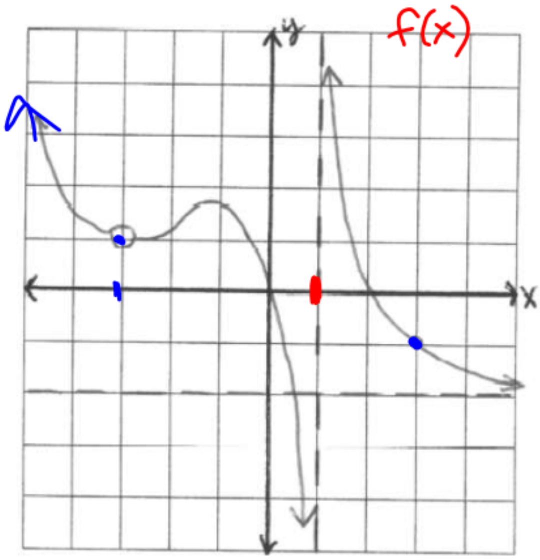
24. $\lim_{x\to 1} f(x) =$ DNE

25. $\lim_{x\to -3} f(x) = 1$

26. $\lim_{x\to 3} f(x) = -1$

27. $\lim_{x\to -\infty} f(x) = \infty$

28. $\lim_{x\to \infty} f(x) = -2$



29) $y = 3x^2 - 2x + 1$

Tangent @ $x = -1$

Point:

$y = 3(-1)^2 - 2(-1) + 1$

$y = 6$

$(-1, 6)$

Slope: (Derivative)

$y' = 6x - 2$

$y' = 6(-1) - 2$

$y' = -8$

$$\boxed{y - 6 = -8(x+1)}$$

29) b) Equation of normal line @ $x = -1$

$$\boxed{y - 6 = \frac{1}{8}(x+1)}$$

Negative reciprocal slope of tangent.

20

30) $s(t) = 2t^3 - 15t^2 + 24t - 10$

$v(t) = 6t^2 - 30t + 24$

$a(t) = 12t - 30$

a) $s(3) = 2(3)^3 - 15(3)^2 + 24(3) - 10$

$= \boxed{-19}$

b) $v(3) = 6(3)^2 - 30(3) + 24$

$= \boxed{-12}$

c) $a(3) = 12(3) - 30$

$= \boxed{6}$

d) Slowing down b/c $v(3)$ and $a(3)$ have different signs.

e) At rest when $v(t) = 0$

$0 = 6t^2 - 30t + 24$

$= 6(t^2 - 5t + 4)$

$= 6(t-4)(t-1)$

$t = 4 \quad | \quad t = 1$

$t = 4$
$t = 1$

f) Moving → when $v(t)$ is positive

$\oplus \quad - \quad \oplus$

Test 0 | 1 | Test 2 | 4 | Test 5

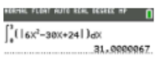
Moving Right $[0, 1) \cup (4, \infty)$

g) Used to be tough b/c we had to calculate distance over intervals of left and right motion and then add them.

$$\text{Distance} = \int_a^b |v(t)| \, dt$$

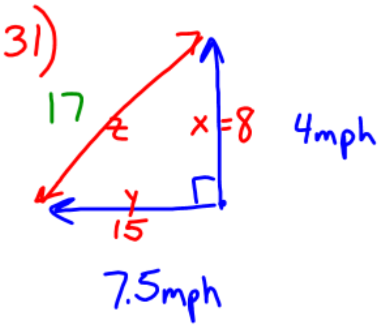
absolute value to get rid of + and - velocity

$$\text{Distance} = \int_0^3 |6t^2 - 30t + 24| \, dt$$



$\text{Dist} = 31$

31)



$8^2 + 15^2 = z^2$

$64 + 225 = z^2$

$289 = z^2$

$17 = z$

Have:

$x = 8 \quad \frac{dx}{dt} = 4$

$y = 15$

$z = 17 \quad \frac{dy}{dt} = 7.5$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(8)(4) + 2(15)(7.5) = 2(17)\frac{dz}{dt}$

$64 + 225 = 34\frac{dz}{dt}$

$\frac{289}{34} = \frac{34\frac{dz}{dt}}{34}$

$8.5 = \frac{dz}{dt}$

8.5 mph

22

32) $x^2 - xy = y^2 + 1$

$y = 1$ QI

Slope

$\frac{d}{dx}\left(x^2 - xy = y^2 + 1\right)$

$2x - \left(x\frac{dy}{dx} + 1y\right) = 2y\frac{dy}{dx}$

$2x - x\frac{dy}{dx} - y = 2y\frac{dy}{dx}$

$2(2) - 2\frac{dy}{dx} - 1 = 2(1)\frac{dy}{dx}$

$3 - 2\frac{dy}{dx} = 2\frac{dy}{dx}$

$+2\frac{dy}{dx} \quad +2\frac{dy}{dx}$

$\frac{3}{4} = \frac{4\frac{dy}{dx}}{4}$

$\frac{dy}{dx} = \frac{3}{4}$

$$\boxed{y - 1 = \frac{3}{4}(x - 2)}$$

point:

$x^2 - x(1) = 1^2 + 1$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ QI

$x = -1$ reject not QI

$(2, 1)$ point